Detecting oil and gas using elastic waves

Presenter: Erick Mubai

University of the Witwatersrand,

School of Computer Science and Applied Mathematics,

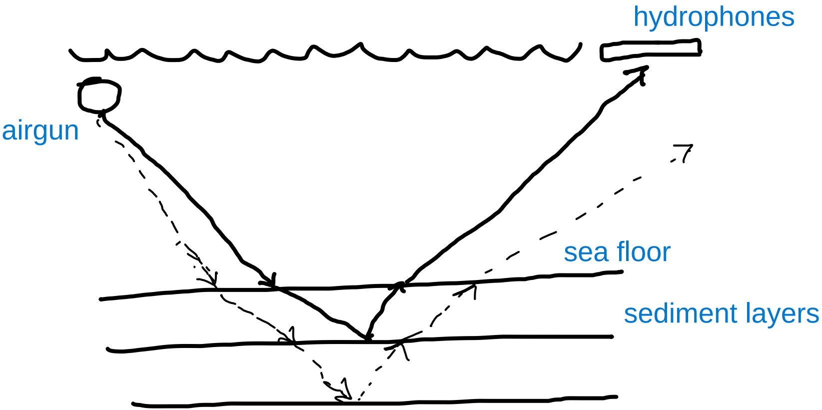
erick.mubai@students.wits.ac.za

February 1, 2022

Using sound waves to explore the seafloor

- Engineers explore the seafloor using sound waves.
- They look for layers of the seafloor (salt domes) that can trap oil.
- The aim of the study is to find out how the reflected and refracted sound from the seafloor can be used investigate the physical properties of the layers of the seafloor.
- Hence determine the possibility of oil and gas underneath the seafloor.

How sound waves can detect oil?



Elasticity in review

- The displacement vector

$$\underline{u} = (u_x, u_y, u_z)$$

- For an isotropic material the generalized Hooke's law is

$$\tau_{ik} = \lambda\theta\delta_{ik} + 2\mu E_{ik}$$

where

$$\theta = \nabla \cdot \underline{u}, \qquad E_{ik} = \frac{1}{2}\left(\frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i}\right).$$

- Cauchy's first law of motion

$$\rho \frac{Dv_i}{Dt} = \tau_{ki,i} + \rho F_i$$

where

$$v_i = \frac{\partial u_i}{\partial t}.$$

- Substituting the stress tensor we have

$$\rho \frac{\partial^2 u_i}{\partial t^2} = (\lambda + \mu) \nabla\theta + \mu\nabla^2 u_i + \rho F_i.$$

Elastic body waves

If we consider a displacement dependent on the y coordinate and time:

$$u_x = u_x(y,t), \qquad u_y = u_y(y,t), \qquad u_z = u_z(y,t),$$

we get the wave equations

$$\frac{\partial^2 u_x}{\partial t^2} - \frac{\mu}{\rho}\frac{\partial^2 u_x}{\partial y^2} = 0,$$

$$\frac{\partial^2 u_y}{\partial t^2} - \frac{\lambda + 2\mu}{\rho}\frac{\partial^2 u_y}{\partial y^2} = 0,$$

$$\frac{\partial^2 u_z}{\partial t^2} - \frac{\mu}{\rho}\frac{\partial^2 u_z}{\partial y^2} = 0.$$

Elastic body waves

- For one wave, the displacement is in the direction of the propagation and the speed of the wave is $v_p = \sqrt{\frac{\lambda+2\mu}{\rho}}$.
- For the other two waves, the displacement is perpendicular to the propagation of the wave and the speed of the wave is $v_s = \sqrt{\frac{\mu}{\rho}}$.

Model